the $\Delta T=\frac{1}{2}$ rule. In this way we get

$$\frac{R(K^+\to\pi^+\pi^0)}{R(K_1^0\to 2\pi)} = f_{K^+\to\eta^0\pi^+}{}^2\left(\frac{\delta g_{\pi NN}}{g_{\pi NN}}\right)^2\left(\frac{g_{\pi NN}^2/4\pi}{g_{\eta NN}^2/4\pi}\right)\Big/ f_{K_1^0\to 2\pi}{}^2, \quad (12)$$

where we have used expression (8) for $G_{\eta\pi}$. $f_{K^+\to\eta^0\pi^+}$ and $f_{K_1^0\to 2\pi}$ are the weak-coupling constants for the decays $K^+\to\eta^0+\pi^+$ and $K_1^0\to 2\pi$, respectively, both of which are allowed by a pure $\Delta T=\frac{1}{2}$ rule. But $\eta^0\pi^+$ is a $T=1$ state while the 2π mode in K_1^0 decay is a $T=0$ state, so that if we take

$$f_{K^+\to\eta^0\pi^+}\approx\sqrt{3}f_{K_1^0\to 2\pi},$$

we get

$$\frac{R(K^+\to\pi^+\pi^0)}{R(K_1^0\to 2\pi)}\approx\frac{1}{444},$$

with $g_{\eta NN}^2/4\pi$ again equal to 2 and $(\delta g/g)\approx 1\%$. This result is unchanged if $\delta g/g\approx 0.7\%$ and $g_{\eta NN}^2/4\pi\approx 1$.

There is now some experimental evidence for the violation of the $\Delta T=\frac{1}{2}$ rule in the 3π decay of K_2^0, and the η can also be responsible for such a violation if we consider the sequence

$$K_2^0\to\eta^0\to 3\pi.$$

However, in the absence of any workable procedure to estimate the strength of the weak vertex $K_2^0\to\eta^0$, we do not give any numerical estimate.

That we have been able to correlate so many different processes through the η meson is a consequence of the quantum numbers 0^{-+}, $T=0$ assigned to the η meson.

Lastly let us consider the total decay rate for $K_2^0\to\pi^+\pi^-\pi^0$ as given by Eq. (7). This can be calculated provided that we know $G_{K\pi}$. One can approximately fix $G_{K\pi}$ if one assumes that the $\Sigma^-\to n+\pi^-$ is dominated by the K pole. Then

$$\Gamma(\Sigma^-\to n+\pi^-)=2\left(\frac{g_{\Sigma NK}^2}{4\pi}\right)G_{K\pi}^2\frac{P_\Sigma}{[(m_K/m_\pi)^2-1]^2}, \quad (13)$$

where

$$P_\Sigma=\frac{(\Sigma\pm N)^2-\pi^2}{2\Sigma^2}\left[\left(\frac{\Sigma^2-N^2+\pi^2}{2\Sigma}\right)^2-\pi^2\right]^{1/2};$$

the $\pm$ correspond to the cases of scalar and pseudoscalar $K\Sigma N$ coupling, respectively. Eliminating $G_{K\pi}$ between (7) and (13) and using[27] $R(\Sigma^-\to n+\pi^-)=0.6\times10^{10}$ sec^{-1} and $\lambda/4\pi\approx -0.15$, we find for the pseudoscalar coupling constant $g_{\Sigma NK}^2/4\pi$ the values 3 to 1.5 according as[28] $R(K_2^0\to\pi^+\pi^-\pi^0)=1.5\times10^6$ sec^{-1} or 3×10^6 sec^{-1}. For scalar $K\Sigma N$ coupling,

$$g_{\Sigma NK}^2/4\pi\approx 0.03.$$

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Equivalence of the Brysk Approximation and the Determinantal Method*

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It is shown that the first-order approximations, for central potential scattering, of Brysk and of the determinantal method are equivalent.

In a recent paper[1] Brysk has presented a new approximation for scattering from a potential. He obtains this approximation by iterating on the asymptotic expression for the scattered wave in an asymptotically valid equation for the exact wave function. His result, for a spherically symmetric potential, is

$$\tan\delta_l=\frac{-k\int_0^\infty r^2dr\; j_l^2(kr)U(r)}{1-k\int_0^\infty r^2dr\; j_l(kr)n_l(kr)U(r)}, \quad (1)$$

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where $U(r)\hbar^2/2m$ is the scattering potential, and $j_l(kr)$ and $n_l(kr)$ are the usual spherical Bessel and Neumann functions.

The purpose of this note is to point out that this result is completely equivalent to the lowest order approximation in the determinantal method[2] based on the Fredholm solution for an integral equation.[3] This method considers the determinant

$$D(E)=\det\left(\frac{E-H_0-V}{E-H_0}\right). \tag{2}$$

Since $D(E)$ has poles at the unperturbed energies, E_{0k}, and equals unity for $V=0$, it may be written in the form

$$D(E)=1+\sum_{E_{0k}} r_k/(E-E_{0k}). \tag{3}$$

An expansion of $D(E)$ in powers of V shows that

$$\begin{aligned} r_k=&-\langle E_{0k}|V|E_{0k}\rangle\\ &+\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n!}\sum_{E_{01},E_{02},\cdots E_{0n}}\prod_{i=1}^{n}\left(\frac{1}{E_{0k}-E_{0i}}\right)\\ &\times\begin{vmatrix}\langle E_{0k}|V|E_{0k}\rangle & \langle E_{0k}|V|E_{01}\rangle\cdots & \langle E_{0k}|V|E_{0n}\rangle\\ \langle E_{01}|V|E_{0k}\rangle & \langle E_{01}|V|E_{01}\rangle\cdots & \vdots\\ \vdots & \vdots & \vdots\\ \langle E_{0n}|V|E_{0k}\rangle & \langle E_{0n}|V|E_{01}\rangle\cdots & \langle E_{0n}|V|E_{0n}\rangle\end{vmatrix}\end{aligned} \tag{4}$$

where the $|E_{0k}\rangle$ are eigenstates of H_0. This series converges for all strengths of the potential provided

$$\int_0 dr\, rV(r)<\infty$$

and

$$\int^{\infty} dr\, V(r)<\infty. \tag{5}$$

For the case of scattering, one can show[2] that

$$\pi r(E)\cot\delta(E)=1+P\int_0^{\infty}\frac{r(E')dE'}{E-E'}, \tag{6}$$

where we have now taken the unperturbed energy levels to form a continuum and

$$r_k=r(E_{0k})dE_{0k}. \tag{7}$$

Hence, one has the exact expression

$$\tan\delta_l=\pi r_l(E)\bigg/\left[1+P\int\frac{r_l(E')}{E-E'}dE'\right], \tag{8}$$

where we have evaluated explicitly for the lth angular momentum state.

For the lowest order approximation, one takes the first term in the expansion (4) for r_k. This gives

$$\begin{aligned} r(E)&=-\langle E_{0k}|V|E_{0k}\rangle m/\hbar^2 k\\ &=-\frac{1}{\pi}k\int_0^{\infty}r^2dr\, j_l^2(kr)U(r),\end{aligned} \tag{9}$$

with our normalization. Hence,

$$\tan\delta_l\approx\frac{-k\int_0^{\infty}r^2dr\, j_l^2(kr)U(r)}{1-\frac{1}{\pi}P\int\frac{dE'}{E-E'}k'\int_0^{\infty}r^2dr\, j_l^2(k'r)U(r)}, \tag{10}$$

where $E'=\hbar^2k'^2/2m$ and $E=\hbar^2k^2/2m$.

The numerator in (10) is the same as that in (1), and is, of course, just the Born approximation. To show that the denominators are the same, we must evaluate the principal part integration in (10) and show that

$$\frac{1}{\pi}P\int_0^{\infty}\frac{dE'}{E-E'}k' j_l^2(k'r)=kj_l(kr)n_l(kr), \tag{11}$$

or, introducing the ordinary Bessel and Neumann functions, and letting $z=kr$, $z'=k'r$, we must show that

$$\frac{2}{\pi}P\int_0^{\infty}dz'\frac{z'}{z'^2-z^2}J_{l+1/2}^2(z')=-J_{l+1/2}(z)N_{l+1/2}(z). \tag{12}$$

But this follows immediately from the relation[4]

$$\begin{aligned}\lim_{\epsilon\to0+}\frac{2}{\pi}\int_0^{\infty}dz'\frac{z'}{z'^2-(z+i\epsilon)^2}J_\mu^2(z')\\ =J_\mu(z)[iJ_\mu(z)-N_\mu(z)].\end{aligned} \tag{13}$$

Hence, we have shown the equivalence of (10) and (1). The expansion (4) then provides a simple systematic method of generating convergent higher order approximations to (1).

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[4] G. N. Watson, *Theory of Bessel Functions* (Cambridge University Press, New York, 1958), p. 429.